Poznan University of Technology

Faculty of Electronics and Telecommunications

Chair of Communication and Computer Networks

Simultaneous Connections Routing in W-S-W Elastic Optical Switches with Limited Number of Connection Rates

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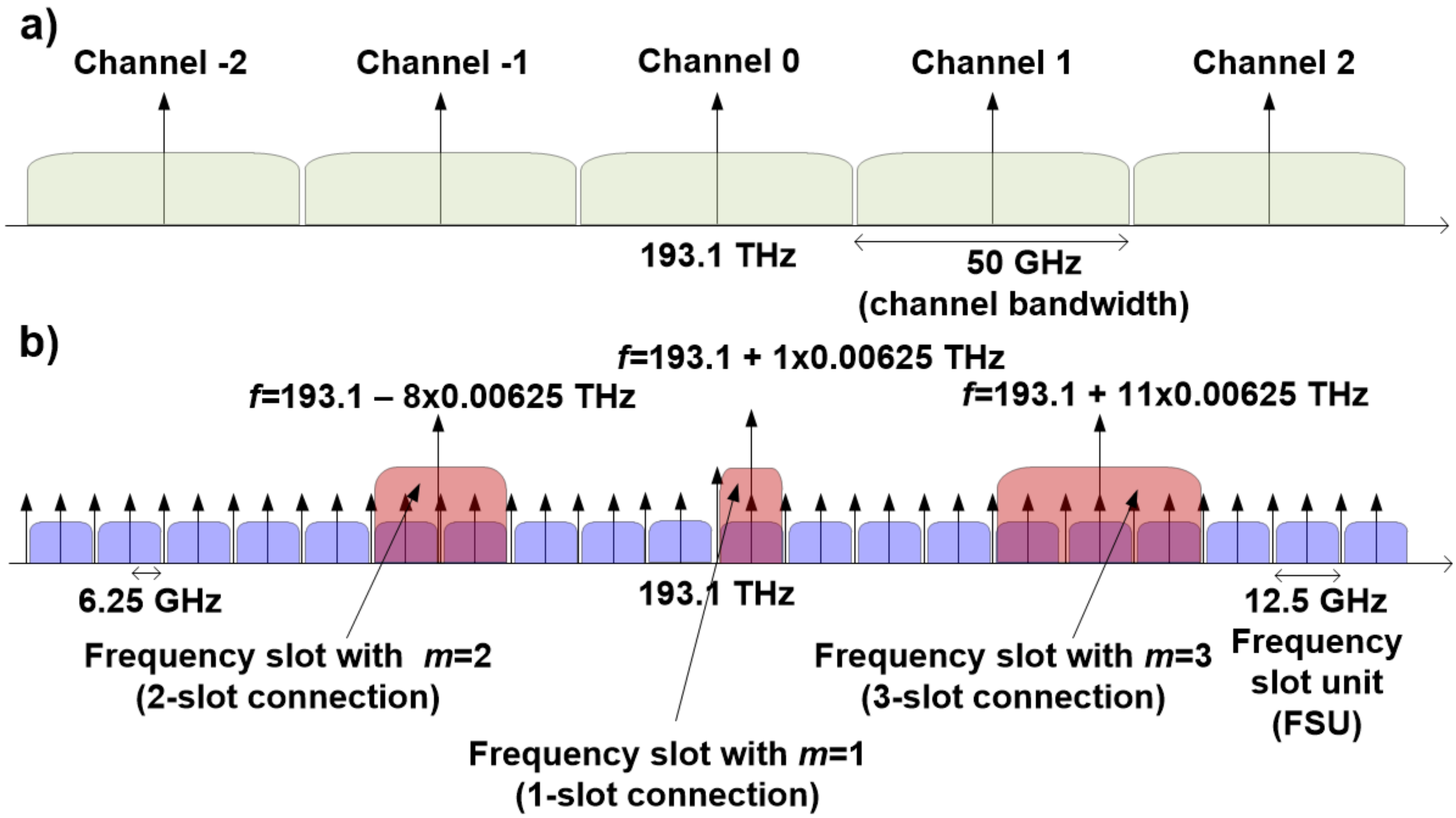
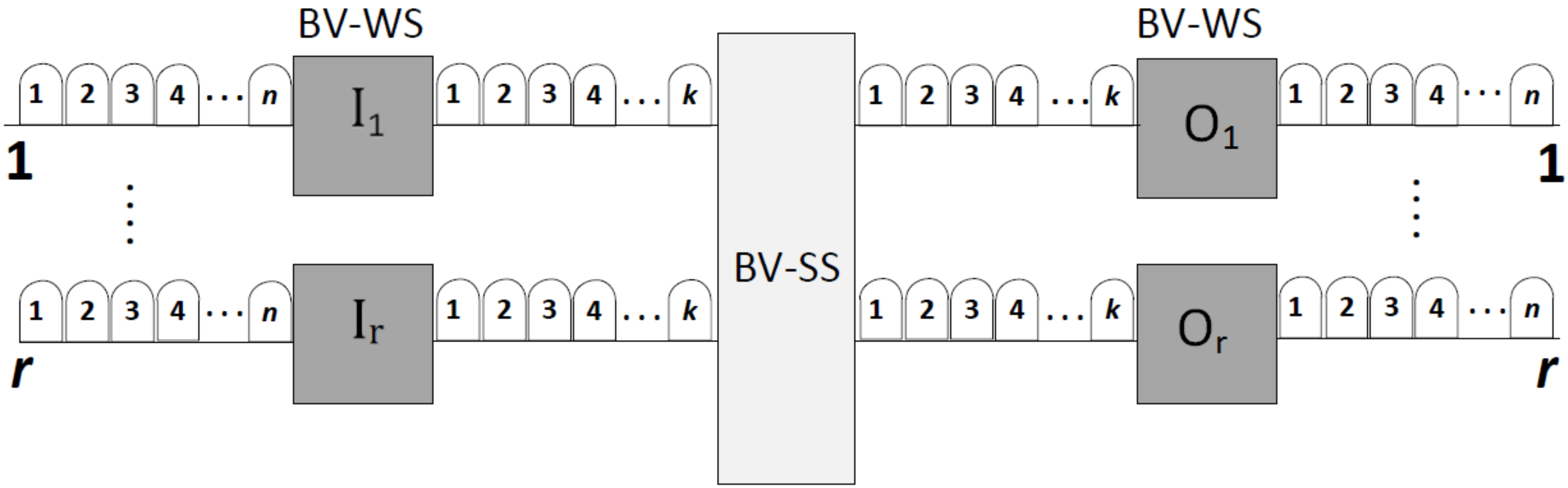
Outline

- Introduction
- Problem Statement
- Model Description
- Control Algorithm
- Theorem 1 and 2
- Example
- Conclusion and Future Work

Elastic Optical Networking

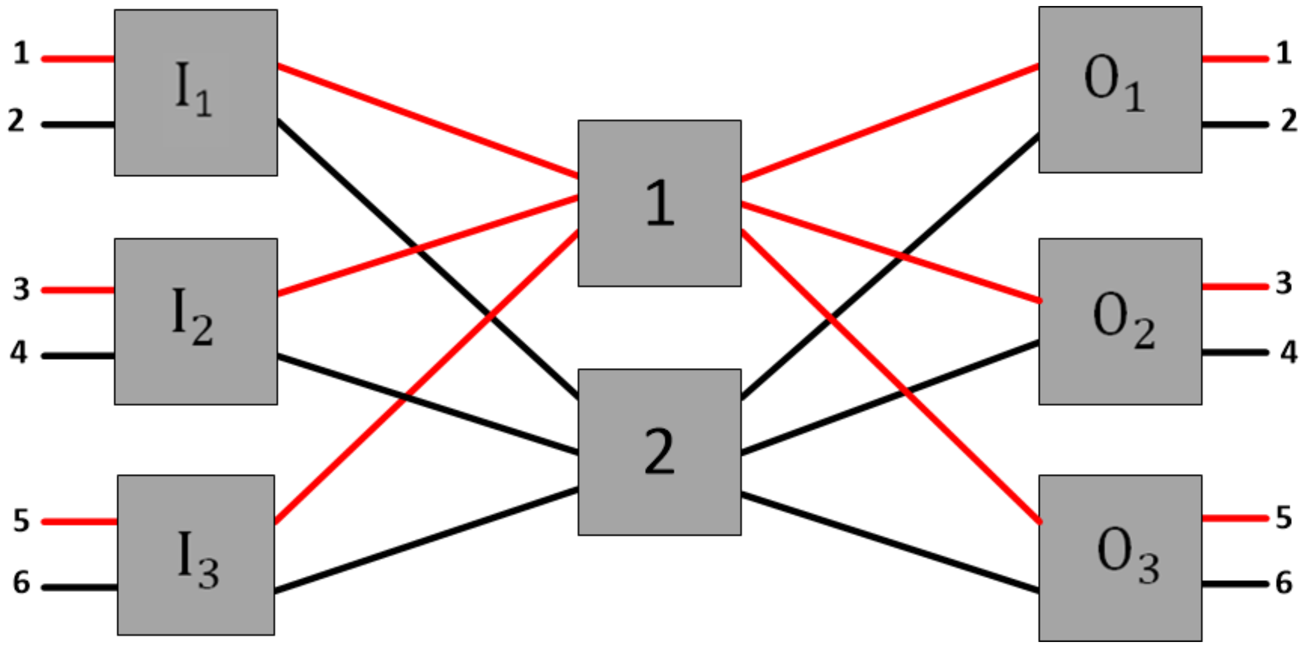


Switching Fabric Architecture



W-S-W (wavelength-space-wavelength) switching fabric.

Three Stage Clos Network



$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 3 & 5 & 2 & 1 & 5 \end{pmatrix}$$

$$\mathbf{H} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$P_1 = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

Three Stage Clos Network

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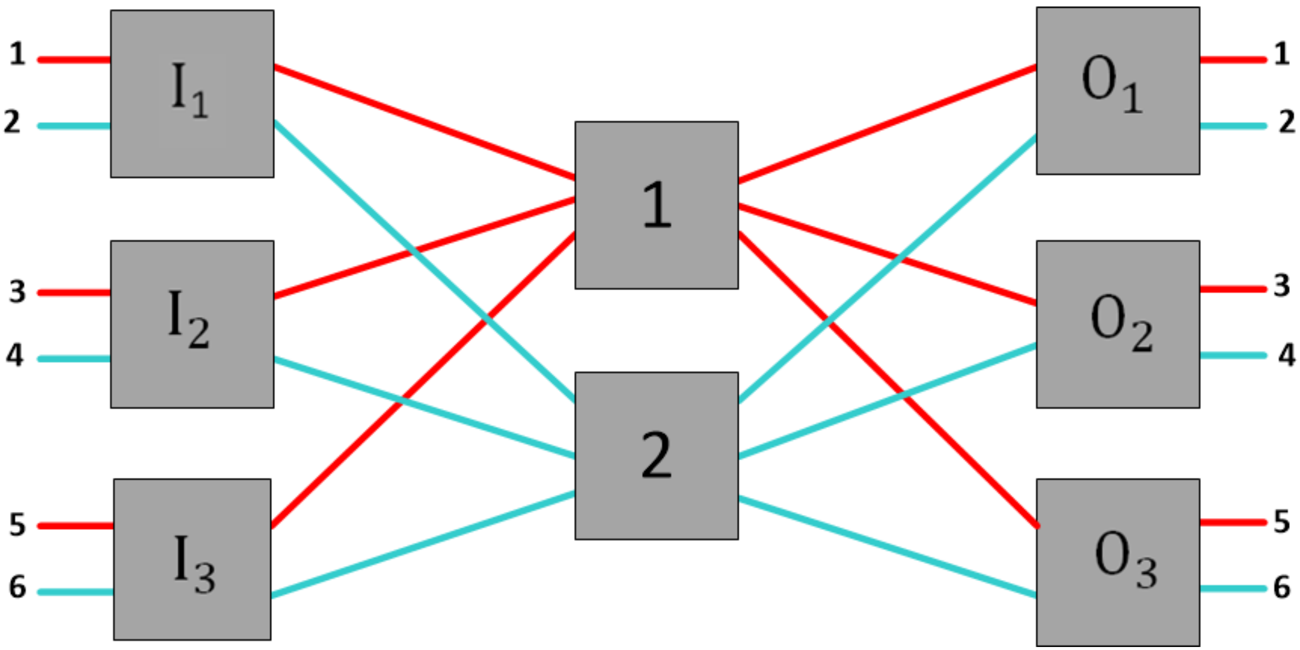
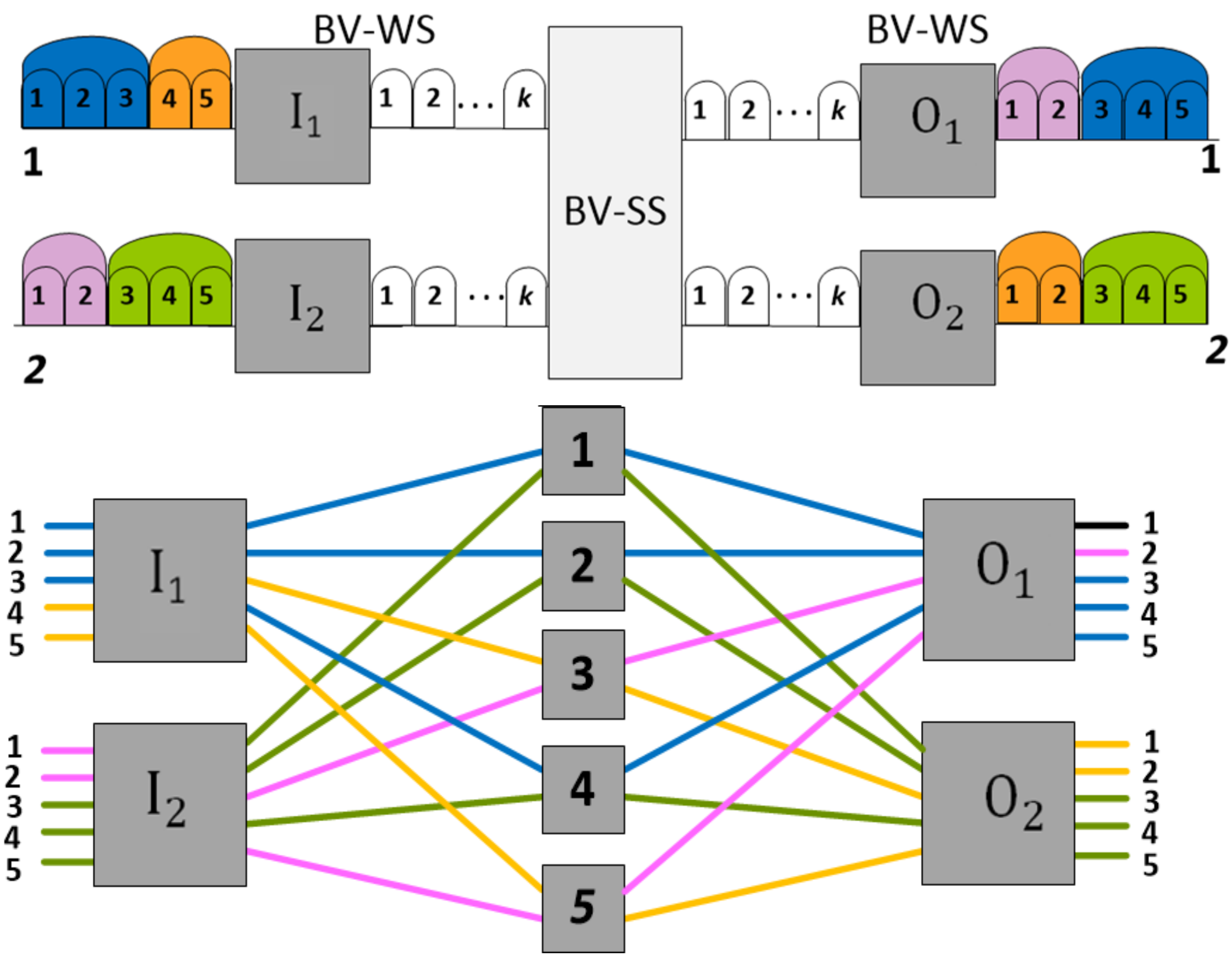


$$\mathbf{H} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$P_1 = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$P_2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

Problem Statement



$$H = \begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}$$

$$P_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$P_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$P_3 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$P_4 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$P_5 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Assumption

- We have a set of compatible connection requests $\mathbb{C}$.
- m-slot connection is denoted by $(I_i[x], O_j[y], m)$.
- Limited number of connection rates:

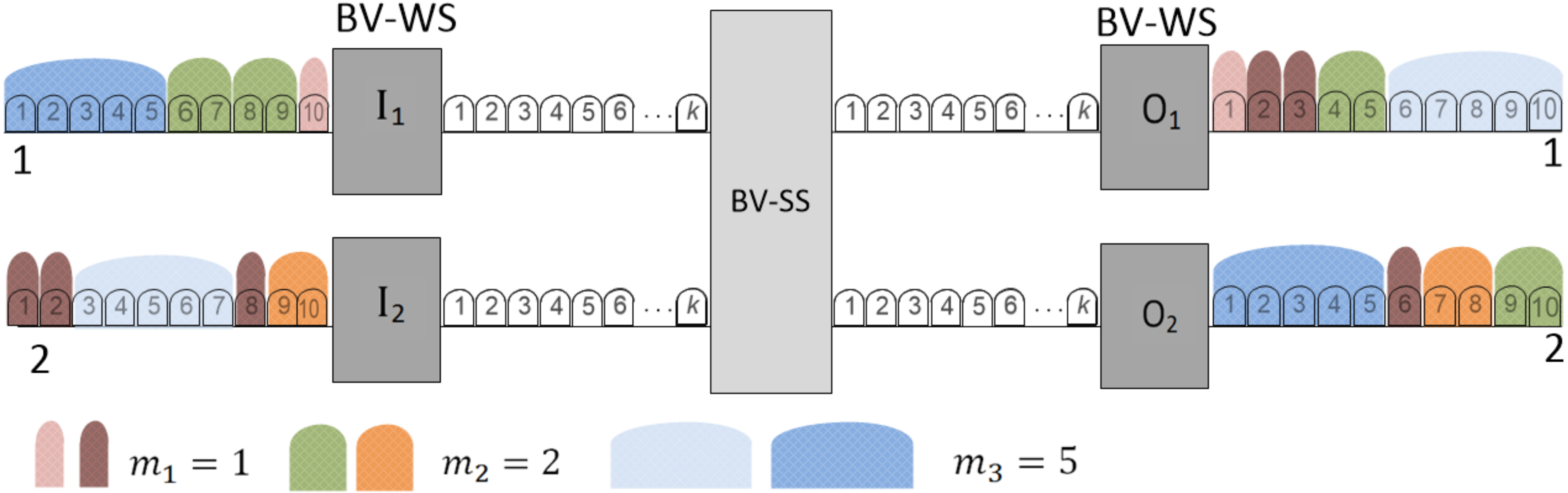
  m_1-slot connections

  m_2-slot connections

  ⋮

  m_z−slot connections

Example of $\mathbb{C}$



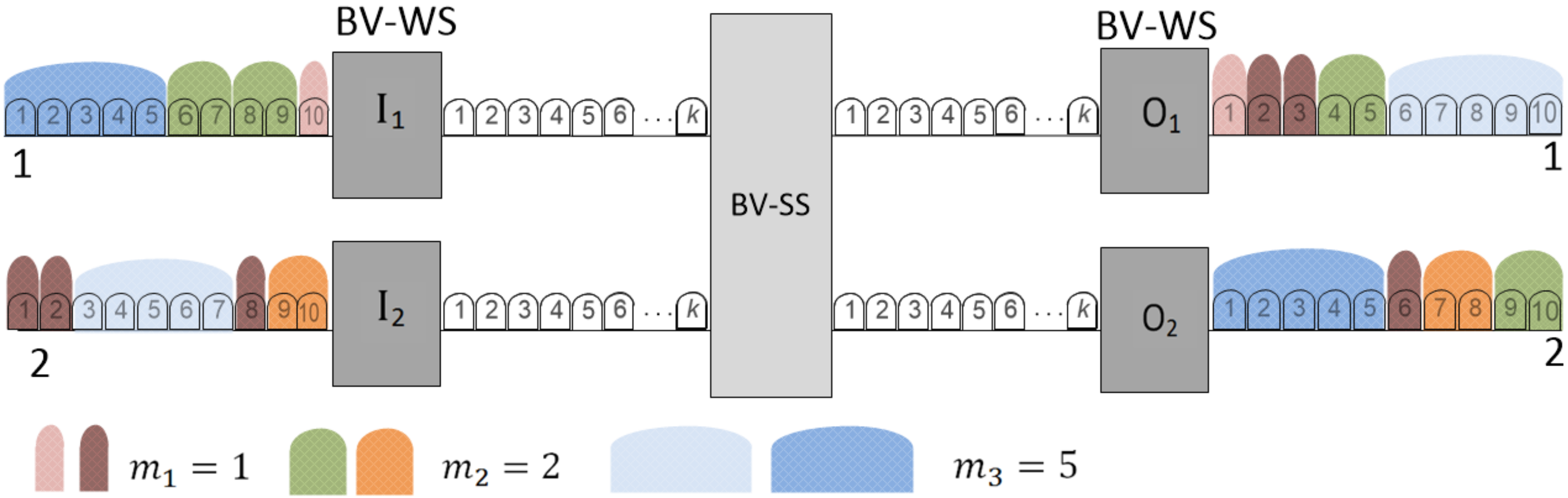
$$\mathbb{C} = \{(I_1[1], O_2[1], 5); \quad (I_2[1], O_1[2], 1);$$
$$(I_1[6], O_1[4], 2); \quad (I_2[2], O_2[6], 1);$$
$$(I_1[8], O_2[9], 2); \quad (I_2[3], O_1[6], 5);$$
$$(I_1[10], O_1[1], 1); \quad (I_2[8], O_1[3], 1);$$
$$(I_2[9], O_2[7], 2)\}.$$

Model Description

- All connections of a given size can be represented as a matrix: $H^{m_x}, 1 \leq x \leq z$.

$$H^{m_x} = \begin{bmatrix} h_{11}^{m_x} & \cdots & h_{1r}^{m_x} \\ \vdots & \ddots & \vdots \\ h_{r1}^{m_x} & \cdots & h_{rr}^{m_x} \end{bmatrix}$$

Model Description



$$H^{m_1} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \quad H^{m_2} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \quad H^{m_3} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Properties of H^{m_x}

- All matrices H^{m_x} have the following properties:
  - $\sum_{j=1}^{r} \sum_{x=1}^{z} (h_{ij}^{m_x} * m_x) = n$
  - $\sum_{i=1}^{r} \sum_{x=1}^{z} (h_{ij}^{m_x} * m_x) = n$
  - If $z = 2, r = 3$

$$H^{m_1} = \begin{bmatrix} h_{11}^{m_1} & h_{12}^{m_1} & h_{13}^{m_1} \\ h_{21}^{m_1} & h_{22}^{m_1} & h_{23}^{m_1} \\ h_{31}^{m_1} & h_{32}^{m_1} & h_{33}^{m_1} \end{bmatrix} \qquad H^{m_2} = \begin{bmatrix} h_{11}^{m_2} & h_{12}^{m_2} & h_{13}^{m_2} \\ h_{21}^{m_2} & h_{22}^{m_2} & h_{23}^{m_2} \\ h_{31}^{m_2} & h_{32}^{m_2} & h_{33}^{m_2} \end{bmatrix}$$

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Control Algorithm

$$H^{m_x} = \begin{bmatrix} h_{11}^{m_x} & h_{12}^{m_x} & h_{13}^{m_x} \\ h_{21}^{m_x} & h_{22}^{m_x} & h_{23}^{m_x} \\ h_{31}^{m_x} & h_{32}^{m_x} & h_{33}^{m_x} \end{bmatrix}$$

$a_1^{m_x}$

$a_2^{m_x}$

$a_3^{m_x}$

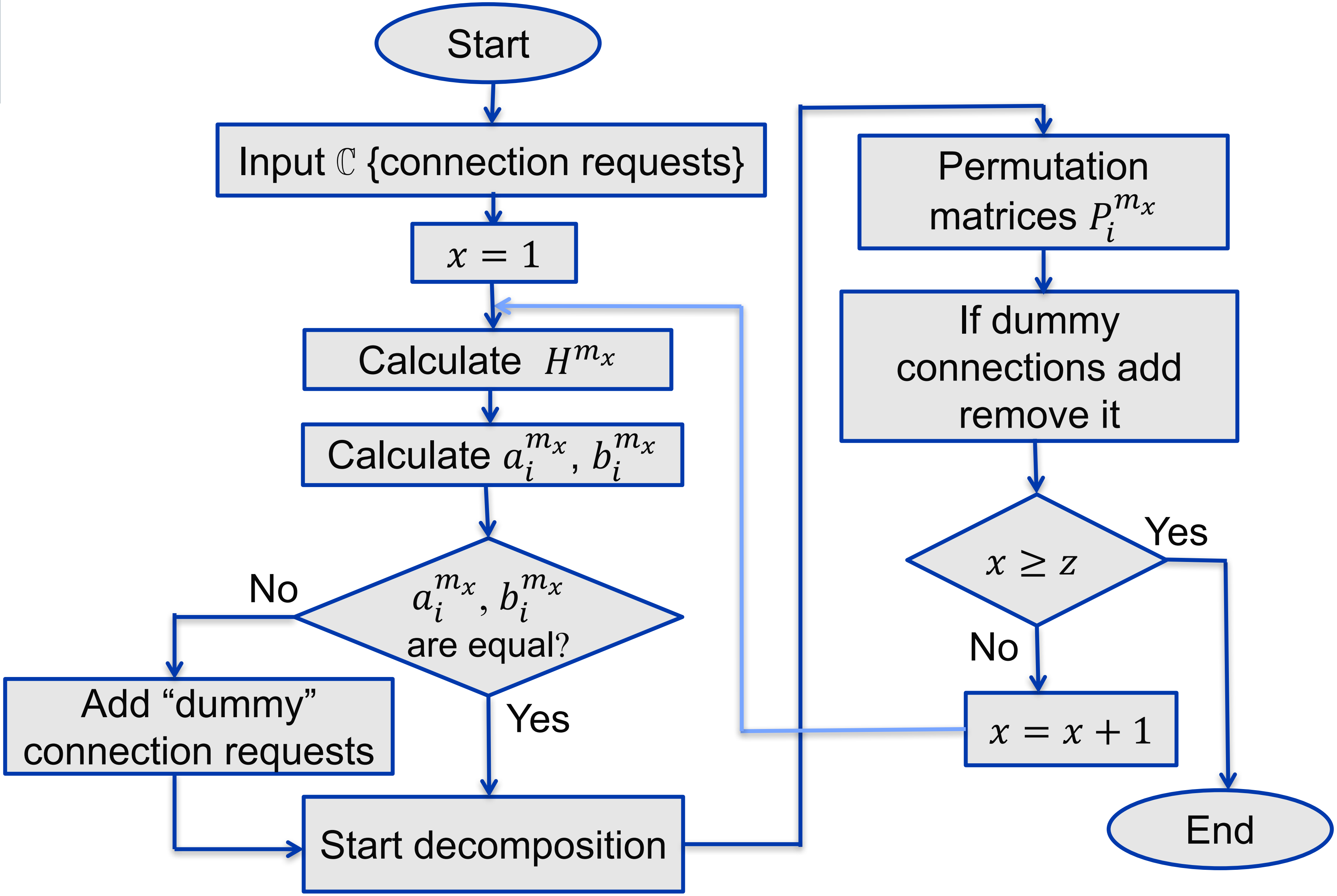
$b_1^{m_x}$ $b_2^{m_x}$ $b_3^{m_x}$

$a_{max}^{m_x}$ $a_{min}^{m_x}$

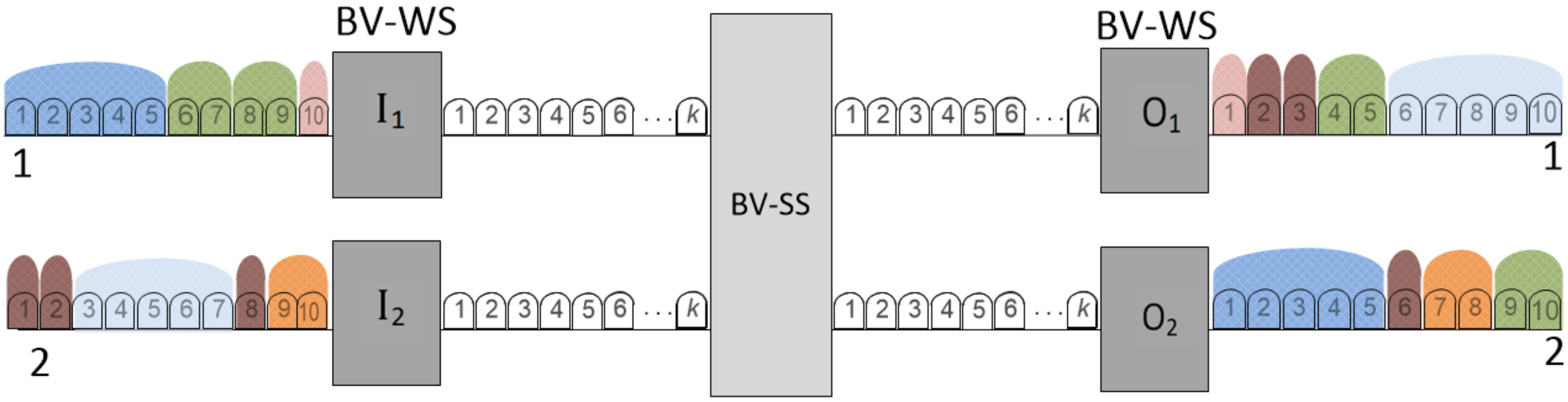
$b_{max}^{m_x}$ $b_{min}^{m_x}$

$$c_{max}^{m_x} = max\{a_{max}^{m_x};\ b_{max}^{m_x}\}$$

$$c_{min}^{m_x} = min\{a_{min}^{m_x};\ b_{min}^{m_x}\}$$


Start
Input ℂ {connection requests}
$x = 1$
Calculate H^{m_x}
Calculate $a_i^{m_x}$, $b_i^{m_x}$
$a_i^{m_x}$, $b_i^{m_x}$ are equal?
No
Yes
Add “dummy” connection requests
Start decomposition
Permutation matrices $P_i^{m_x}$
If dummy connections add remove it
$x \geq z$
Yes
No
$x = x + 1$
End

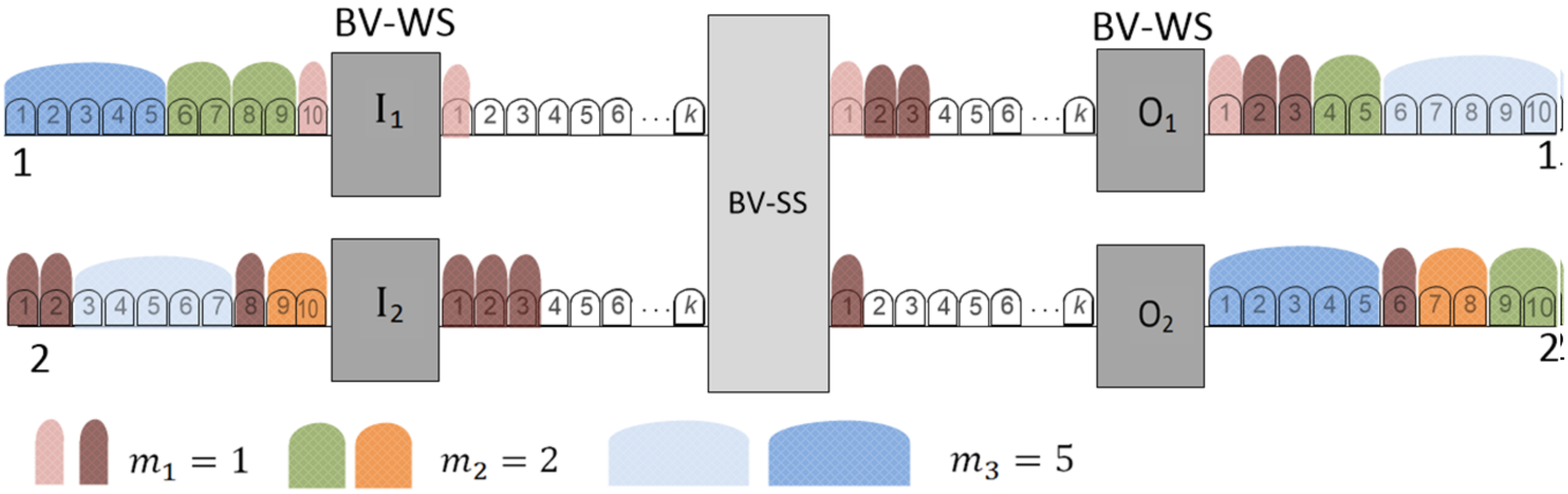
Example



$m_1 = 1$ $m_2 = 2$ $m_3 = 5$

$$H^{m_1} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \quad H^{m_2} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \quad H^{m_3} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

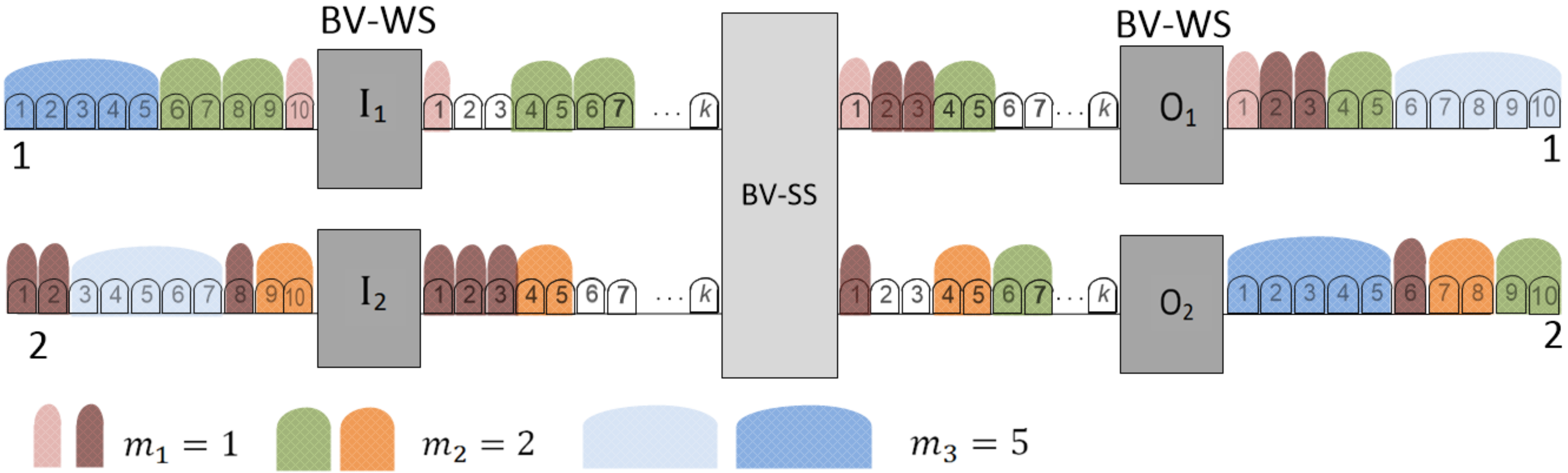
Example



$$H^{m_1} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \longrightarrow H^{m_1} = \begin{bmatrix} 1' & 2 \\ 2 & 1' \end{bmatrix} \longrightarrow P_1^{m_1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$H_1^{m_1} = H^{m_1} - P_1^{m_1} = \begin{bmatrix} 0 & 2' \\ 2' & 0 \end{bmatrix} \longrightarrow P_2^{m_1} = P_3^{m_1} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$$

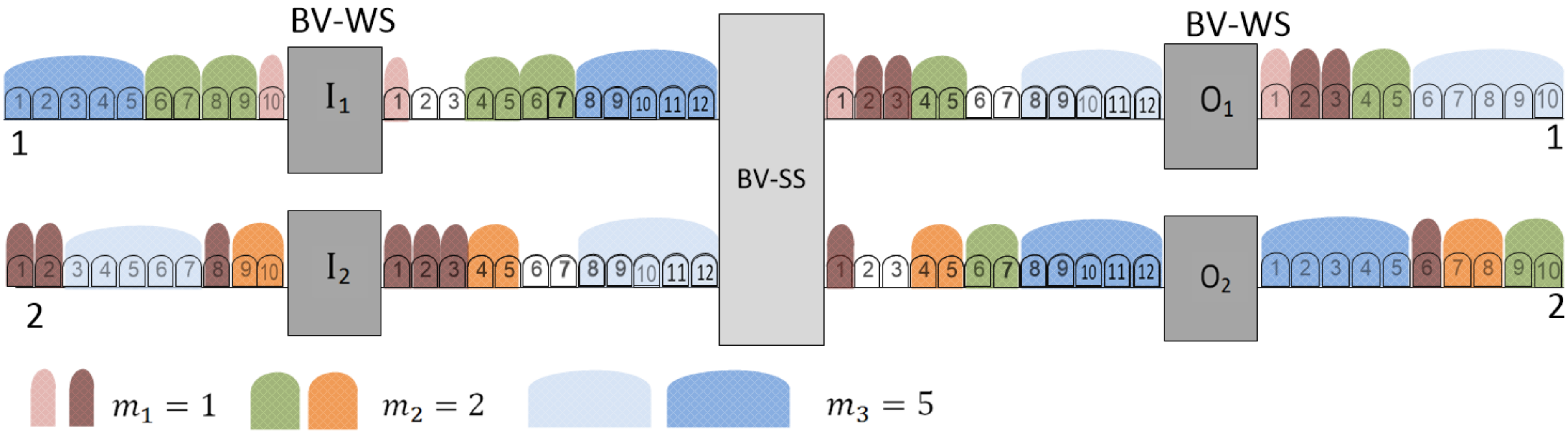
Example



$$H^{m_2} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \longrightarrow H^{m_2} = \begin{bmatrix} 1' & 1 \\ \color{red}{1} & 1' \end{bmatrix} \longrightarrow P_1^{m_2} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$H_1^{m_2} = H^{m_2} - P_1^{m_2} = \begin{bmatrix} 0 & 1' \\ \color{red}{1'} & 0 \end{bmatrix} \longrightarrow P_2^{m_2} = \begin{bmatrix} 0 & 1 \\ \color{red}{0} & 0 \end{bmatrix}$$

Example



$$H^{m_3} = \begin{bmatrix} 0 & 1' \\ 1' & 0 \end{bmatrix} \longrightarrow P_1^{m_3} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Theorem 1

- The WSW1 switching fabric is RNB for m-slot connections, $x \in \{m_x\}$ and $1 \leq x \leq z$, if:

$$k \geq \sum_{x=1}^{z} \left(\left\lfloor \frac{n}{m_x} \right\rfloor * m_x \right)$$

Theorem 2

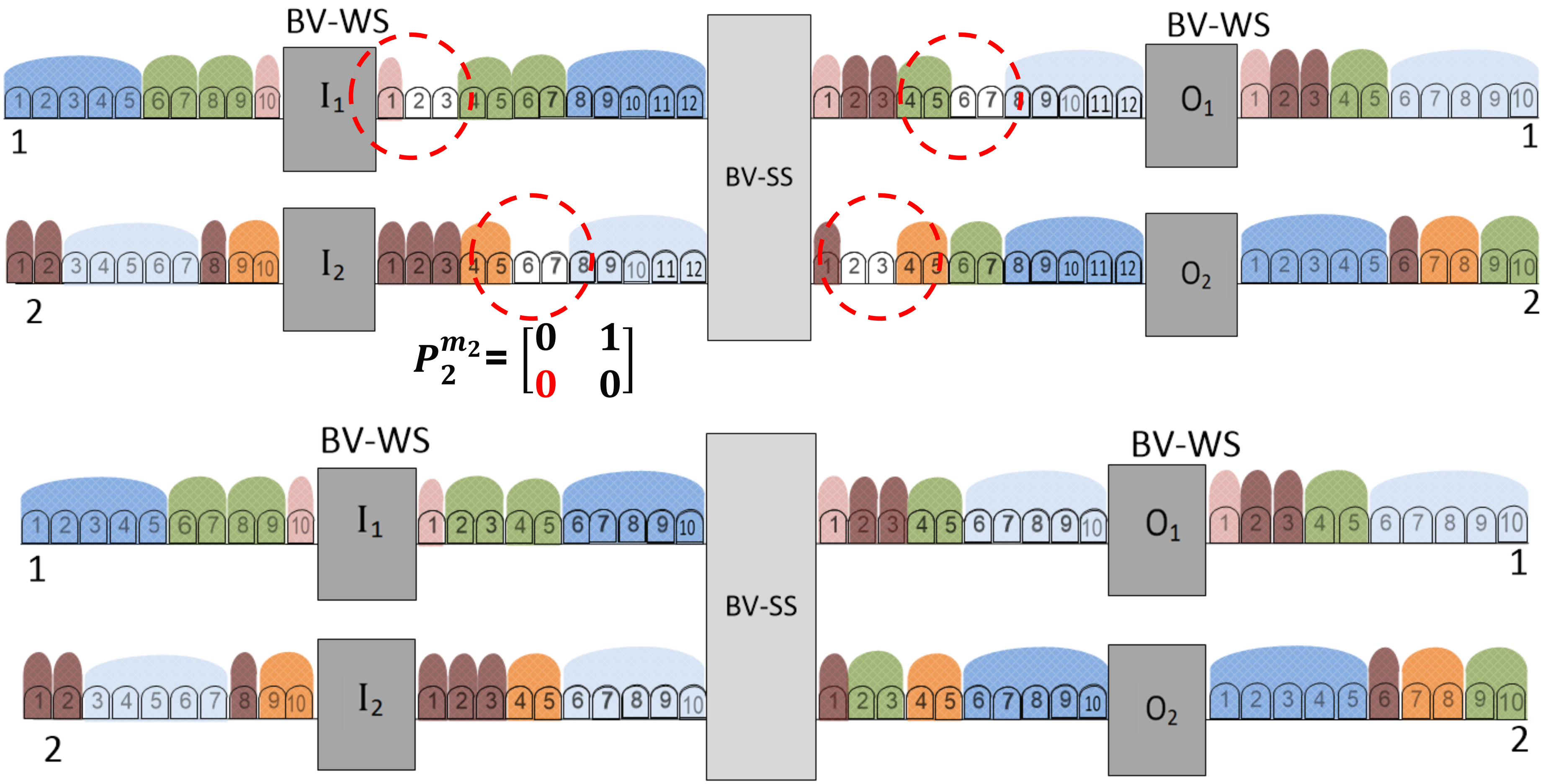
- The WSW1 switching fabric with $r = 2$ is rearangeably nonblocking for m-slot connections, where $m \in \{m_1; m_2\}$, $m_1 < m_2$, $\frac{n}{m_1}$, $\frac{n}{m_2}$, and $\frac{m_2}{m_1}$ are integers, is RNB if and only if:

$$k \geq n$$

Example

$$P_2^{m_1} = P_3^{m_1} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$$

$$P_2^{m_2} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

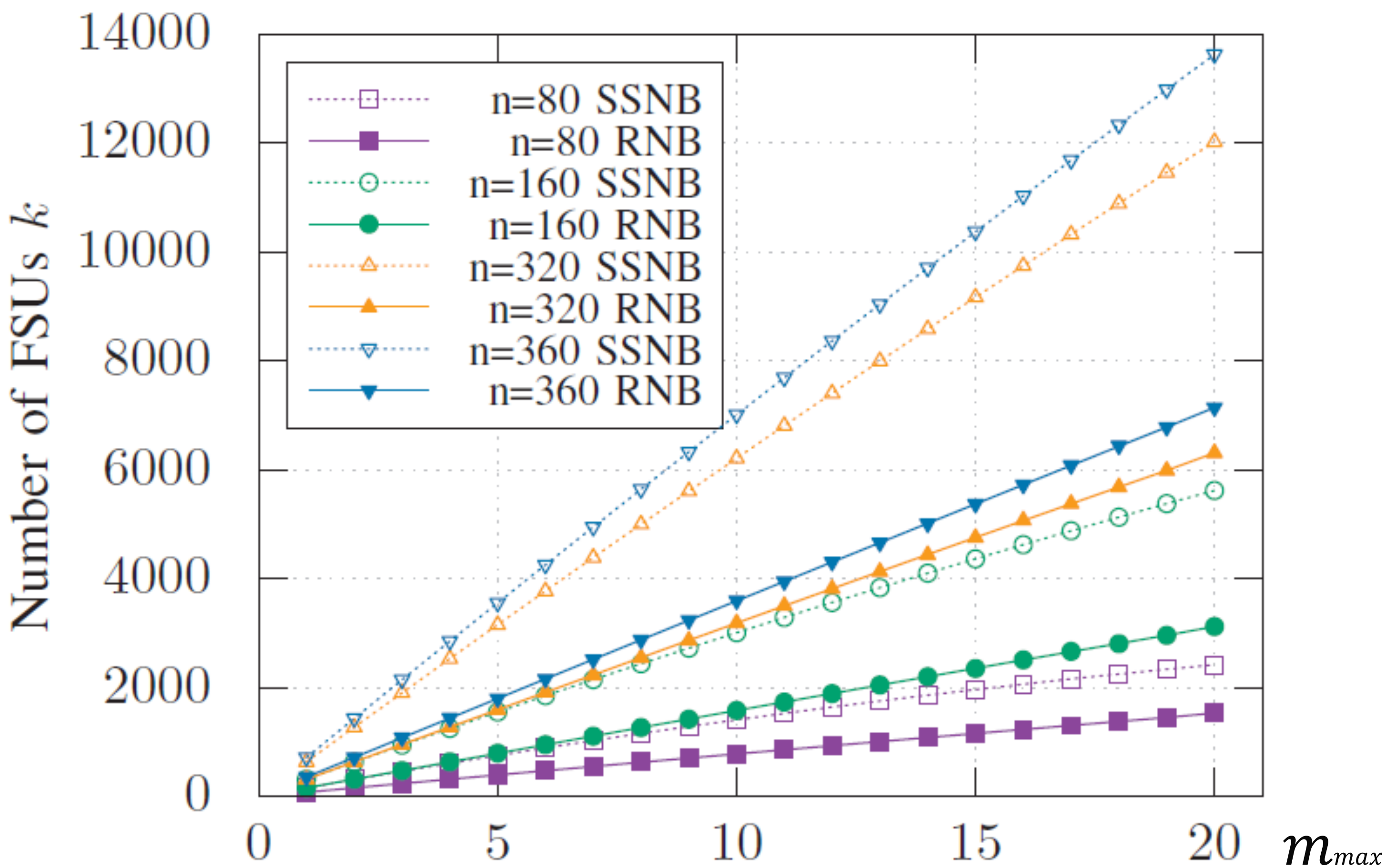


Conclusion and Future Work

- We proposed the control algorithm for simultaneous connection routing in the WSW1 switching fabric.
- For this algorithm, we considered the upper bound for RNB operation in a case when the number of connection rates is limited to z.
- For WSW1 with a limited number of $r = z = 2$, where $\frac{n}{m_1}$, $\frac{n}{m_2}$, and $\frac{m_2}{m_1}$, are integers, we proved the necessary and sufficient conditions for RNB operation.
- Extend case $r = z = 2$ when $\frac{n}{m_1}$, $\frac{n}{m_2}$, and $\frac{m_2}{m_1}$ are not integers.
- Improve upper bound for $r > 2$.

Thank You
Any Questions?

Comparison between the RNB and SSNB conditions



Number of FSUs *k* versus m_{max} for selected *n* in SSNB and RNB WSW1 switching fabrics